

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

--	--	--	--	--	--	--	--	--	--

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

4 MARCH 2019
9 a.m. – 11 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5)** printed pages inclusive of the cover page, formulae sheet.
2. Answer **ALL** four questions in the answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [20 marks]

- (a) Recent statistics suggest that 25% of those who visit Uniqlo website make a purchase. The retailer would like to verify the claim. To do so, she selected a sample of 8 “hits” to her site.
- i) Find the probability that less than 2 “hits” resulted in a purchase. [5 marks]
 - ii) Find the probability that 5 of the “hits” will end up without any purchase. [3 marks]
 - iii) Find the standard deviation on the number of “hits” with a purchase. [2 marks]
- (b) Waiting times to received food after placing an order at the local Subway sandwich shop is on average 60 seconds.
- i) Calculate the probability that a customer waits more than 0.6 minutes. [5 marks]
 - ii) Calculate the probability that a customer waits between 40 and 70 seconds. [5 marks]

Question 2 [20 marks]

- (a) Given that cars passing KLCC tower with the rate of 3 per second starting from 7am,
- i) Find the probability that 6 cars passing KLCC tower for the first 5 seconds. [4 marks]
 - ii) Find the probability that time between 2 cars passing the KLCC tower is less than 1 second. [3 marks]
 - iii) Find the probability that time between 2 cars passing the KLCC tower is between 0.5 seconds and 1.5 seconds. [6 marks]
- (b) A random walk starts from the origin. The chances for the walker to move to the right are 4 times the chances he moves to the left.
- i) Find the probability that the walker is at the position 2 after 8 steps. [4 marks]
 - ii) Find the probability that the walker is at position -3 after 3 steps. [3 marks]

Continued...

Question 3 [30 marks]

(a) Given a continuous random variable X has the following cumulative density function:

$$F(x) = 3x^2 - 2x^3 \quad \text{for } 0 < x \leq 1$$

- i) Find the probability density function. [3 marks]
- ii) Find the moment generating function. [9 marks]

(b) Random variables X and Y have the following function:

$$f(x, y) = \begin{cases} \frac{kx^2}{y} & ; x = 1, 2 ; y = 2, 3, 4 \\ 0 & ; \text{otherwise} \end{cases}$$

- i) Find the value k . [3 marks]
- ii) Determine the marginal pmf for X and Y . [8 marks]
- iii) Find $P(X + Y \leq 4)$. [3 marks]
- iv) Determine whether X and Y independent. Explain. [4 marks]

Question 4 [30 marks]

Consider a Markov chain with state space of $\{1, 2, 3\}$ and transition probability matrix as below.

$$\begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.25 & 0.75 & 0 \\ 0.4 & 0 & 0.6 \end{bmatrix}$$

- (a) Draw the transition diagram. [3 marks]
- (b) Find $P(X_2 = 1 \mid X_0 = 2)$. [7 marks]
- (c) Determine whether state 2 is transient or persistence state. Find the period of state 2. [10 marks]
- (d) Determine whether each of the states to be absorbing or non-absorbing state. [6 marks]
- (d) Let the initial distribution to be 0.4, 0.2 and 0.4. Find the probability of state 2 after 2 steps. [4 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution
$P(X = x) = p^x q^{1-x}$ for $x = 0, 1$
Binomial Probability Distribution
$P(X = x) = {}^n C_x p^x q^{n-x}$ for $x = 0, 1, \dots, n$
Poisson Probability Distribution
$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$ for $x = 0, 1, 2, \dots$
Geometric Probability Distribution
$P(X = x) = pq^{x-1}$ for $x = 1, 2, \dots$
Uniform Probability Distribution
$f(x) = \frac{1}{b-a}$ for $a < x < b$
Exponential Probability Distribution
$f(x) = \lambda e^{-\lambda x}$ for $x > 0$

B. MOMENT

k^{th} Moment about the origin of X:
$\mu'_k = E[X^k]$
k^{th} Moment about the mean of X:
$\mu_k = E[X - \mu]^k$
Moment Generating Function for X:
$M_X(t) = E(e^{tx})$
Derivative of Moment Generating Function for X:
$M_X^{(r)}(t) = E(X^r e^{tx})$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)} \quad \text{where } m \text{ and } n \text{ are both even or both odd.}$$

$$E(X_n) = n(p - q) \quad ; \quad \text{Var}(X_n) = 4npq$$

Gambler's Ruin Problem

$$P_a = \begin{cases} \frac{\left(\frac{q}{p}\right)^a - \left(\frac{q}{p}\right)^c}{1 - \left(\frac{q}{p}\right)^c}, & p \neq q \\ 1 - \frac{a}{c}, & p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = \begin{cases} \frac{1}{q-p} \left[a - c \frac{1 - \left(\frac{q}{p}\right)^a}{1 - \left(\frac{q}{p}\right)^c} \right], & p \neq q \\ a(c-a), & p = q \end{cases}$$

D. POISSON PROCESS

Pmf for $N(t)$:

$$P\{N(t) = n\} = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$$

Pdf for T_n :

$$f(t) = \lambda e^{-\lambda t}$$

Pdf for S_n :

$$f(t) = \frac{\lambda e^{-\lambda t} (\lambda t)^{n-1}}{(n-1)!}$$